Creating Cultures of Thinking in Math

www.instructionalleadershipteam.com

How might educators create the conditions for students to think deeply about math? How might educators create experiences in math that create curiosity, challenge students to build their math skills, stretch themselves as mathematicians, and meaningfully contribute to the collective understanding of the group? How might educators create the space for students to own their learning in math? How might educators create a culture where students can make their thinking visible, celebrate their mistakes, and feel confident that they can succeed in math? How might educators foster a love of math, and help students see that math is beautiful, creative, and all around us?

The most important thing is the teacher's belief that all students are capable, competent, curious, and like to be challenged.

The 5 Principles of the Modern Mathematics Classroom ~Gerald Aungst (2016)

CONJECTURE

In a traditional mathematics classroom, the primary goal is for students to get the right answers to questions and exercises. In a classroom where conjecture is encouraged, students ask most of the questions, and the answer to a question is very often another question.

COMMUNICATION

In a conventional classroom, communication is primarily one way: the teacher explaining a procedure or algorithm to students. In a problem-oriented classroom, students must learn to communicate frequently about problems and how they solve them.

COLLABORATION

In a traditional classroom, students work alone, and the emphasis is on an individual's skill fluency. Problem solving classrooms are all about the "we".

CHAOS

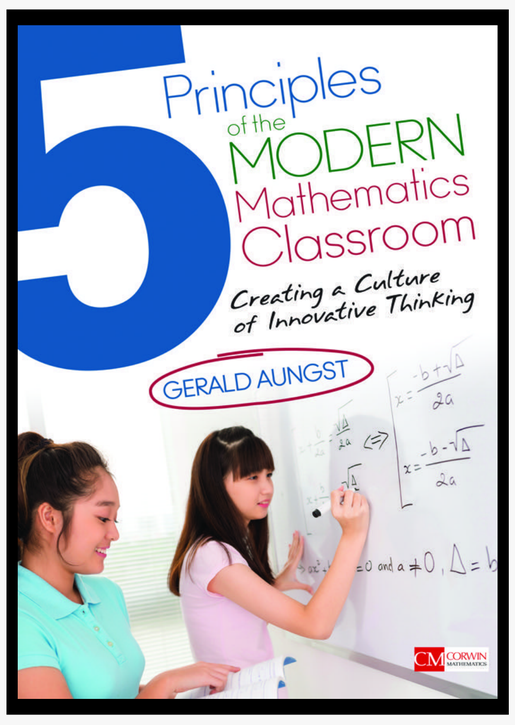
Though this sounds sketchy, it is simply encapsulating the idea that real math work is messy. In a traditional classroom, neatness and order rule the day. Real problems, on the other hand, require experimentation, false starts, mistakes, and corrections, sometimes over and over again.

CELEBRATION

In a traditional classroom, recognition is given for right answers and high grades. In a problem-solving culture, anything that leads towards a solution is celebrated: finding one small step of a complicated problem, thinking of an innovative approach even if it doesn't pan out, or even making a spectacular mistake and asking for help.

"Mathematics instruction is frequently conducted under a false assumption: that mathematics is a fixed, linear sequence of skills. This leaves out what makes mathematicians pursue their subject with passion and drive: the mystery and magic of math. By learning and applying five simple principles, you can inspire students, teach them how to think like innovators, and make them believe they are all "math people." The 5 Principles let you build a classroom culture that emphasizes the most important part of any mathematics curriculum: the processes, practices, and habits of mind."

~ Gerald Aungst, 5 Principles of the Modern Mathematics Classroom: Creating a Culture of Innovative Thinking (2016)



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CONJECTURE

The cure is simple: put the problem first!

"Teachers need to create an environment where mathematical reasoning is essential. We need to immerse students in environments that provide organic opportunities to use and apply math concepts."

"Mathematics is a beautiful, open, creative, and multi-dimensional subject. But school mathematics is often uninspiring, procedural and one-dimensional – it is all about memorizing methods and procedures. One-dimensional mathematics is not the mathematics of the world and it is not the mathematics of our 21st century workplace." ~ Jo Boaler.

Most mathematics textbooks are organized with lessons that teach the steps to solve certain types of problems, which are followed by simple problems students use to practice those steps. Finally, the difficult problems are found at the back of the chapter, and students are aware that they will use the steps they have practiced to solve those problems. But, we know that humans "love mysteries because our brains crave the joy that comes with discovery. [So, instead, why not begin teaching a math concept by] "pos[ing] a challenge so compelling that students are begging you to help them figure it out? " ~Gerald Aungst

Where can we find these challenges?

Find rich, open tasks that give entry points for all learners, while allowing students to extend themselves to high levels (low floor, high ceiling). These challenges can serve as small mathematical inquiries.

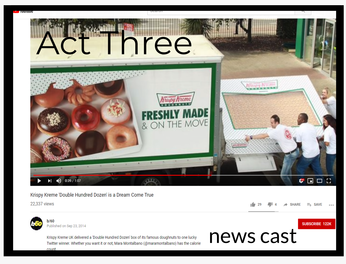
3 ACT TASKS

3 Act Tasks were originally created by Dan Meyer to bring real world, story-based math to students. A **3 Act Task** is a collaborative, open-ended math mystery with three parts.

- **Act One** offers an engaging and perplexing visual (video, image or event). Students are asked what they notice and what wonders they have. As a class, one wonder is chosen to explore.

- In **Act Two** students work in small collaborative groups to determine estimations that they know are too high and too low for the solution to the wonder. They also determine what information they will need to solve the challenge. In this act, students are offered a small amount of new information to consider. Students make their thinking visible and are expected to come to a consensus.

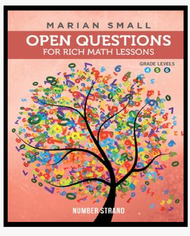
- In **Act Three**, the answer is revealed. Students could continue to inquire and explore some of the other wonders that emerged. This is an opportunity for cross-curricular explorations.



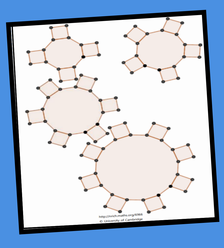
DR. MARIAN SMALL'S OPEN TASKS

Dr. Small believes that "one question can meet the needs of many learners because the question is not overly tight and so benefits a broader range of students." She defines a good open task as one that leads to rich, mathematical conversations and has the ability to stretch all learners. Her books **Open Questions for Rich Math Lessons** are organized by grade and math strand and include questions to help students get started, work on it, and consolidate their understanding.

- There were more than 15 kids in a class.
- Each one paid MORE THAN an $10 but LESS THAN $25 for the field trip.
- How much are you sure is TOO LOW?
- How much is TOO HIGH?
- How much might have been collected?
- Explain your thinking.



"It is better to solve one problem five different ways, than to solve five problems one way."
~George Polya



If you colored one tooth on each cog, which pairs of cogs let the colored tooth go into every 'gap' on the other cog?

Which pairs do not let this happen? Why?

Can you explain how to determine which pairs will work, and why?

JO BOALER'S YOUCUBED OPEN TASKS (https://www.youcubed.org/tasks/)

"Let's consider the work of a mathematician: She first has to pose an important problem, then map out a mathematical approach, she will probably collaborate with others on ideas, and engage in ... a zig-zagging process of conjecturing, refining counter examples and proving. She has to form a mathematical model, apply methods, draw diagrams, connect ideas, reason about connections and communicate in different forms." ~Jo Boaler

Jo Boaler believes that math should be visual and that students should be given opportunities to think about math in different ways: with numbers, with pictures and with words.

It is important for students to engage in productive struggle (where they feel challenged, but not frustrated) when learning math, because when students are struggling and making mistakes, their brains are forming more neural connections.

If 6 cats can kill 6 rats in 6 minutes, how many will be needed to kill 100 rats in 50 minutes?

PETER LILJEDAHL NUMERACY TASKS

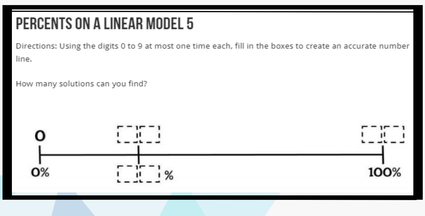
(http://www.peterliljedahl.com/teachers/good-problem)

(http://www.peterliljedahl.com/teachers/numeracy-tasks)

Liljedahl's Tips for helping students find success:
- give instructions verbally
- group students randomly and group students differently each day
- give students the opportunity to work on vertical, non- permanent surfaces (like whiteboards or windows)
- teachers answer student questions with questions of their own.
- encourage students to consult other groups if they become stuck

OPEN MIDDLE MATH (http://www.openmiddle.com/)

Open middle problems provide students with opportunities to reason with evidence and come to consensus with a group. There is often more than one way to solve the problem and while it might be easy to get an answer, it is often more challenging to get an optimum answer. While they may appear to be simple and procedural in nature, the problems are more challenging and complex when you start to solve them.



"...five suggestions that can work to open mathematics tasks and increase their potential for learning: Open up the task so that there are multiple methods, pathways, and representations. Include inquiry opportunities. Ask the problem before teaching the method. Add a visual component and ask students how they see the mathematics. Extend the task to make it lower floor and higher ceiling. Ask students to convince and reason; be skeptical."
~Jo Boaler, Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching

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COMMUNICATION

When we share our thinking, we create a deeper understanding for ourselves and for others!

> "If you can't explain it simply, you don't understand it well enough."
>
> ~Albert Einstein

Math is about solving problems, and problems are solved through communication as students explain, argue, defend, critique, and discuss their mathematical ideas. Deep math learning takes place when students are talking to each other, building on each other's ideas, and writing and communicating with words, as well as with math symbols.

When teacher talk dominates whole-class discussion, students tend to rely on the teachers to be the experts. When students are at the center of their own learning, they discover they can work out their own solutions and learn from other students.

Through listening, talking and writing about mathematics, students are prompted to organize, re-organize and consolidate their mathematical thinking and understanding, as well as analyze, evaluate and build on the mathematical thinking and strategies of others.

To truly consolidate understanding, Jo Boaler believes that students should also be asked to engage in three levels of convincing: convince yourself, convince a friend, and convince a skeptic.

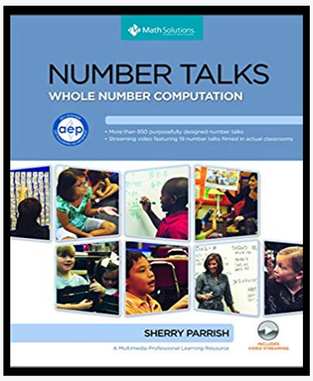
> "Teaching is listening, learning is talking."
>
> *~Deborah Meier*

How Can We Create Opportunities for Students to Communicate Their Thinking in Math?

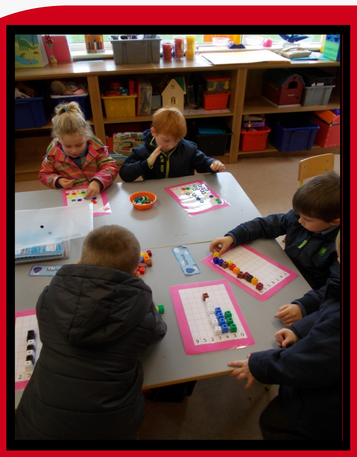
NUMBER TALKS

Number Talks were created by Sherry Parrish to give students daily opportunities to communicate their thinking in math. Number sense is built over time, and can be developed every day of the school year and in any grade, regardless of the concept you are working on. Daily Number Talks can be done as a 5-15 minute routine to help students build their number sense, their mental math skills, and to help them make sense of their own thinking so they can share their thinking with others. This will increase the number of strategies they can use to think about math. Number talks are not teaching opportunities. Teachers use this time to make student thinking visible. The key elements to number talks are a de-emphasis on speed and right answers, an added emphasis on process and communication, and seeing value in understanding multiple strategies shared by peers.

How to do a Number Talk (typically done with 3-4 questions that are connected)**:**
(1) The teacher writes a simple problem down on the board, and students solve it mentally. The difference is that the students aren't just looking for the answer: they're trying to find as many different ways to solve the problem as they can. Students use silent signals to communicate this to the teacher. **(2)** When they are ready, students surface all of their answers and then are asked to defend one answer by sharing one strategy they used. **(3)** The teacher's main role in a number talk is to listen to student strategies, and record the thinking to make it visible so other students can understand what is being shared.



Consider recording all of your student's strategies on an anchor chart so they can refer to them in the future.



NUMERACY STATIONS

Numeracy stations are opportunities to bring PLAY and FUN into the math classroom. They provide students with the opportunity to work collaboratively as they practice and apply the skills and strategies through games, technology, and problem solving. While students are engaged in purposeful stations, teachers can conference with students either individually, or in small flexible groups to check in with students to see how they are progressing with their goals, and to assess where they are with their learning. This time can also be spent scaffolding skills.

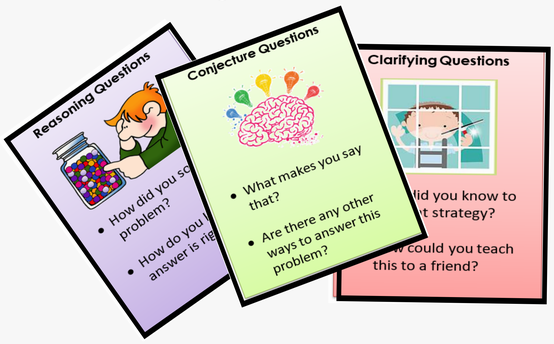
Math stations should be designed to:

- provide meaningful practice to help students develop their curricular competencies in all math content areas
- include a variety of activities that are differentiated to meet the needs of each student
- provide choice for students, as well as opportunites for students to work on different skills at the same time
- change regularly according to the learning goals and needs of students
- give students ongoing opportunities to communicate and share their mathematical thinking with their peers

ESTIMATION 180

Estimation 180 challenges were created by Andrew Stadel and can be found at www.estimation180.com

Estimation challenges help students improve both their number sense and their problem solving skills. Encourage students to first think on their own about a value they know is too high and too low. Then have students work in groups to come to a consensus about what value is too high and too low, still within reason. Next, have groups share out and decide as a class which values are unreasonable. This creates guidelines to help students estimate reasonably and feel safe sharing their estimations. Finally, students think individually, then discuss in groups to determine their best guess for the estimation challenge.



SAY SOMETHING CARDS

Marnie Birkeland and Lindsey Schroeder created Numeracy Say Something Cards as prompts to facilitate student conversations in math. They can be used to help scaffold the way students communicate with each other when discussing their mathematical thinking. A primary and intermediate version of the cards are available here: http://bit.ly/MathSaySomething

Created by Marnie Birkeland & Lindsey Schroeder (Instructional Leadership Team)

COLLABORATION

But isn't that cheating?

"Collaborative learning teaches students to rely on each other in problem solving instead of going to their teacher."

~Gerald Aungst

Think Avengers, not the Lone Ranger! In the real world, problems are not solved by individuals acting alone. Big picture innovation depends on our ability to share ideas and build on each other's thinking.

Solving problems solo should be a rare event in the classroom, and should only be practiced **AFTER** students are confident in their problem solving abilities in groups.

What are the benefits of collaborating in math?

- Allows students to see and talk about all the different ways they see math, and the different strategies they use. In doing so, it allows them to see math in ways they may not have thought of on their own. They come to understand that math is a creative subject and that there are many different ways to think about numbers.
- It gives students the opportunity to connect and build on each others' thinking.
- Students learn how to disagree respectfully. Collaboration develops students' skills to create convincing mathematical arguments as they defend and explain their thinking to others.
- It helps students negotiate coming to a consensus in a group.
- Students become risk-takers as they persevere in problem solving, change their thinking, and try new ways when their current methods aren't working.

It is also important to:

CO-CREATE CRITERIA: WHAT DO GOOD COLLABORATORS IN MATH DO? **students can use this criteria to set personal learning intentions prior to collaborative learning**

When should math be collaborative?

Daily Number Talks

Students share their thinking to build the collective knowledge of the group

Rich, Open Tasks

Students attempt to find multiple solutions to tasks that have no one right answer

Math Inquiry

Students attempt to answer their own mathematical questions (ie: 3-Act Tasks)

Math Council

Students gather to present mathematical arguments and ask questions to push the collective thinking

Sharing Circle

Students gather with the teacher to make their thinking visible as they struggle to make sense of difficult concepts

Partner Share & Feedback

Students share their thinking, reflect on their thinking with partners, and give each other kind, specific, and useful feedback

Numeracy Stations

Students share their thinking as they play

Should math always be collaborative?

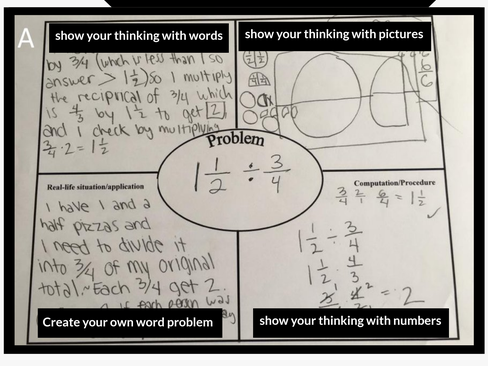
It is important to allow students to build understanding together, and it is equally important to give students quiet, reflective time to make sense of their own thinking. Independent practice is not a bad thing when it is in service of helping students to grow their numerical skills. When the focus is on deep learning, rather than completing the work, each student may need different amounts of practice.

Formative Assessment Opportunities

CONSIDER THIS

Ask students to choose some questions and make their thinking visible numerically, with pictures, written words and/or orally.

This might be done with the A.N.I.E. , a piece of paper folded to create 4 boxes (see below), with the app 'explain everything,' and in a teacher-student conference.



Most great learning happens in groups. Collaboration is the stuff of growth. ~Sir Ken Robinson

Alone we can do so little. Together we can do so much ~Helen Keller

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CHAOS

Real math is messy!

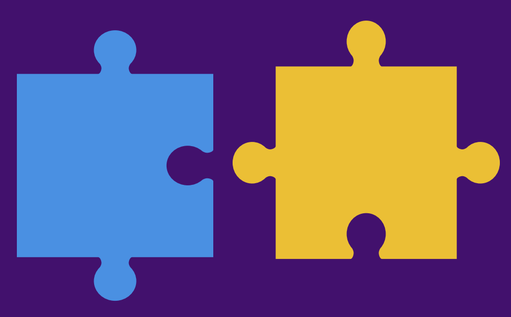
Hiebert and Grouws (2007) find only two features of teaching that have enough evidence to confidently say they have a significant impact on student conceptual understanding of mathematics.

1) when teachers and students explicitly attend to **concepts** (as opposed to merely discrete facts and skills).
2) when students **struggle** with important mathematics.

"Chaos doesn't mean that students are not on task, it means that students are involved in the messiness of real problem solving. Edison famously said, "I have not failed, I've just found 10,000 ways that won't work." One of the biggest problems with math instruction is that we show students the one way that someone else figured out would work, and then ask them to rehearse it to perfection." ~Gerald Aungst

"Much of formal education... feels like learning the pieces of a picture puzzle that never gets put together, or learning about the puzzle without being able to touch the piece."

~ David Perkins, founding member of Project Zero



Remember that the goal of solving a math problem is not for students to get good at solving ***that*** problem. It is for them to get better at solving ***all*** problems.

When we struggle, our brains grow and we learn!

It is important to let students struggle. **We, as teachers, need to get out of the way**. "To create classrooms that cultivate these skills, the formula is simple: provide students with challenging problems, empower them with the skills and resources they need to solve those problems, then get out of their way." Ginsburg, 2014.

DESTRUCTIVE STRUGGLE = leads to frustration, makes learning goals feel hazy and out of reach, feels fruitless/pointless, leaves students feeling abandoned and on their own, and creates a sense of inadequacy. This happens when the challenge is outside of a students' zone of proximal development.

PRODUCTIVE STRUGGLE = leads to understanding, makes learning goals feel attainable and effort seem worthwhile, yields results, leads students to feelings of empowerment and efficacy, and creates a sense of hope. This is the type of struggle where students begin to understand their own thought processes and to develop the kinds of thinking that lead to better solutions. Don't be too quick to bail them out!

"Instead of pointing students directly to the correct solution path when their first attempts fail, initially focus on what worked, then on why their solution did not ultimately succeed." ~Gerald Aungst

"Every time a student makes a mistake in math, they grow a synapse."

~Jo Boaler

Instead of using leading questions, use thought-provoking questions like: What did you do well? What makes you think that? What kinds of errors could have caused this? Can you think of another strategy you could use? How could you think about that differently?

CELEBRATION

Validate effort and process, not answers

"If we believe that growth is possible, and that we can always learn, change, and improve, then a failure simply becomes a 'not yet.' When we validate effort (instead of intelligence) then students can develop a growth mindset, focusing on steady improvement, and believing that everyone can learn, it just takes work to get there." ~Gerald Aungst

Students should be asked to set learning intentions for math. To do this, consider creating a learning map that shows students how they can develop their curricular competencies in math. These learning maps can be created for each unit, or can be created with a year long focus. Learning maps should be created with student-friendly language so students can interact with it regularly. See an example of a math learning map here: bit.ly/MathLearningMap

When students feel like they have succeeded or grown with their learning intention, they can be asked to reflect and select evidence to show how they have met that goal. Then, they can determine their next steps in learning.

It is the learning that should be celebrated. When students are asked to own their own learning and know where they are with their learning, they will get to know themselves as learners and their confidence will grow.

Students can also celebrate each other for improving as a collaborator, or for trying a new strategy, or for sharing a convincing mathematical argument, etc.

"Mistakes lead to good places so if you make a mistake, take it as a step up the learning ladder"

~Jessica, age 9

Tips for Celebrating in Math

1. Focus on the learning
2. Showcase process & skill
3. Normalize struggle
4. Promote Growth Mindset

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MATH CONGRESS

Math Congress is a mathematics instructional strategy developed by Fosnot and Dolk (2002). The purpose of the congress is to support the development of mathematicians in a learning community. This is a great way to bring all 5 Principles of the Modern Mathematics Classroom to life!

Steps to Creating a Successful Math Congress:

1. Activate student thinking around a skill you want them to develop with a provocation. This could be a 3 Act Task, or any other challenge that will inspire curiosity. Throughout the challenge, which will most likely take one class, students will be thinking about the inquiry question individually, in small groups, and ultimately, to the whole class in a sharing circle as they make visible what they notice, think and wonder.

2. Challenge students to collaborate, communicate, make their thinking visible and come to consensus. Sort students into partners and give them a set of two or three rich, open tasks to consider. These tasks should have multiple, correct answers and should be low floor, high ceiling so all students have an entry point. In partners, students will each have a poster and a 2 different coloured felt pens so they cannot erase their thinking. Students will make their thinking visible on their poster, and if they make mistakes, they can show how they learned from their mistakes by recording **C.M.T. (Changed My Thinking).** This is a way to celebrate the process of learning. If they get stuck they can consult neighbouring teams for help. Depending on the open tasks, this phase can last approximately 2-4 days.

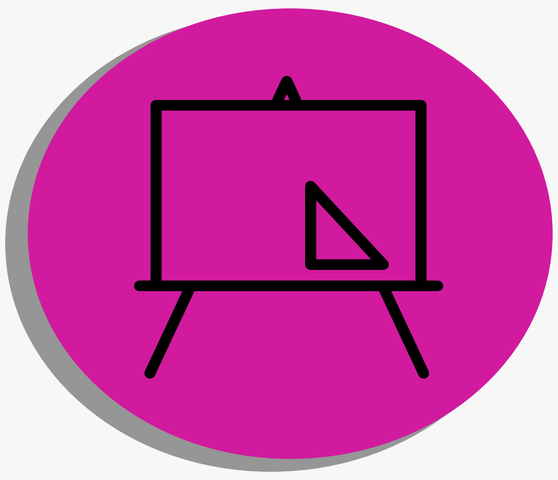
3. Facilitate small group discussions. While students are working on the open tasks, teachers can pull smaller groups of students into a sharing circle to discuss where they are with their thinking so far. This is an opportunity to collectively build further knowledge, to surface misconceptions, to scaffold learning, to introduce any concepts they may need, and to make student thinking visible so students can learn new strategies from each other. This also allows teachers time to assess students. This can be done like a mini Number Talk.

4. Peer-to-Peer feedback. Each team connects with another team so they can give each other feedback. Teachers refer to previously co-created criteria for what makes feedback kind, specific and useful in mathematics. Teachers might choose to give a specific focus for the feedback. Students can use this time to give each other ideas they can use to build on their thinking or make their thinking more clear.

5. Math Council. Math Council is a structure that allows students to communicate, share, and think about their understanding. During Math Council students are asked to bring their posters (showing their thinking about the open tasks) to the circle. Each team may be at a different point, but at minimum, each team will have a mathematical argument for one of the open tasks ready to share.

Math Council is not about showing each team's thinking, instead, it focuses the whole class on the thinking from two or three randomly selected teams. This helps to deepen each student's mathematical learning. The expectation is that everyone must contribute to the conversations during Math Council to develop the collective understanding of the class.

MATH COUNCIL IS LESS A PRESENTATION AND MORE OF A CONVERSATION! Learners gather in a circle for Math Council and selected teams will take turns defending their thinking about their open task(s) by presenting convincing mathematical arguments. Council members are expected to ask clarifying questions, provide feedback, and share their own thinking and strategies. During this process, the teacher might want to offer sentence stems for council members to use to begin the conversation. Teachers will also be documenting conversations and nudging the thinking with their own questions.



It is also important to:

✓ **CO-CREATE CRITERIA: WHAT MAKES A MATHEMATICAL ARGUMENT CONVINCING?**

✓ **CO-CREATE CRITERIA: HOW CAN I MAKE MY MATHEMATICAL THINKING CLEAR AND VISIBLE?**

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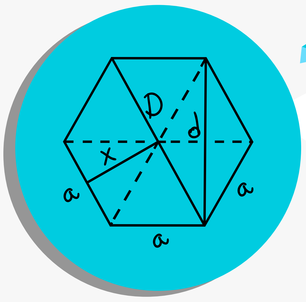
Putting it all Together



Possible Learning Sequence

With all of these principles in mind, and with all of these different ideas and ways to learn math (including what you already do well), sometimes it is difficult to imagine how it all might fit together.

Digital resources available at:
http://bit.ly/MathThinkingBlock

It is all about finding a balance. This will most likely not be a linear process as you make decisions to move at the pace of learning.

Here are some suggestions:

MATH CONGRESS

Pre-Assessment

Before you begin a new component in math, uncover each student's level of understanding.

Daily Math

Build number sense each day with 5-15 min. Number Talks, Estimation180, Which One Doesn't Belong, etc.

Learning Map

Students can use a learning map to help them set clear learning intentions and reflect throughout.

Provoke Thinking

Start with a challenge or mystery to provoke thinking and inspire wondering (ie: 3-Act Task).

Collaborative Open Tasks

In partners, have students select open tasks and make their thinking visible on large posters or vertical non-permanent surfaces.

Teacher Conferencing

Students make their thinking visible in a sharing circle with the teacher as they discuss the open tasks.

Peer Feedback

Teams meet to give each other kind, specific, and useful feedback. Students then use the feedback to improve their mathematical arguments.

Math Council

Some randomly chosen teams share and defend their mathematical arguments for the open tasks. Peers sit in a circle and ask questions.

Numeracy Stations

Students practice their skills by playing with math games, using technology, and/or meeting with the teacher.

Individual Assessment

Each student will show their personal understanding of math concepts with the A.N.I.E., hamburger-hot dog-chip, independent practice, a quiz, etc.

Reflection

Students select evidence of success for their learning intentions, determine next steps, and discuss how their thinking has changed.

SOURCES

Gerald Aungst, 5 Principles of the Modern Mathematics Classroom: Creating a Culture of Innovative Thinking; Jo Boaler, Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching; Educators at the Opal School Museum of Play; Dr. Marion Small; Sherry Parrish; Peter Liljedahl; The Ontario Ministry of Education 2005, CBS Communication in Mathematics; Hiebert, J., & Grouws, D.A. (2007); and educators from the Central Okanagan Public Schools.

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ILT Curated Math Resources:
http://bit.ly/ILTMathResources